Statistics / Module 9 - Probability

Topic 1: Union and intersection of finite sets
Problem 1: Given A = {1, 3, 5} and B = {3, 4, 6}, find A ⎕ B and A ⎕ B.
Answer: A ⎕ B = {1, 3, 4, 5, 6} (all elements in A or B). A ⎕ B = {3} (elements common to both).

Problem 2: For sets C = {2, 4, 6, 8} and D = {4, 8, 10}, compute C ⎕ D and C ⎕ D.
Answer: C ⎕ D = {2, 4, 6, 8, 10} (all elements in C or D). C ⎕ D = {4, 8} (common elements).

Topic 2: Determining a sample space and outcomes for an event: Experiment involving a single selection
Problem 1: A die is rolled once. List the sample space and the event of rolling an even number.
Answer: Sample space = {1, 2, 3, 4, 5, 6}. Event (even number) = {2, 4, 6}.

Problem 2: A letter is chosen from {A, B, C}. List the sample space and the event of choosing a vowel.
Answer: Sample space = {A, B, C}. Event (vowel) = {A} (assuming A is a vowel).

Topic 3: Introduction to the probability of an event
Problem 1: A bag contains 3 red and 2 blue balls. Find the probability of picking a red ball.
Answer: Total balls = 5. Red balls = 3. P(red) = 3/5 = 0.6.

Problem 2: A spinner has 4 equal sections: 1 red, 2 green, 1 blue. Calculate P(green).
Answer: Total sections = 4. Green sections = 2. P(green) = 2/4 = 0.5.

Topic 4: Probability involving one die or choosing from n distinct objects
Problem 1: A die is rolled. Find the probability of rolling a 4.
Answer: Total outcomes = 6. Favorable outcome = 1 (rolling a 4). P(4) = 1/6 ≈ 0.167.

Problem 2: Choose one card from 5 distinct cards (A, B, C, D, E). Find P(card B).
Answer: Total cards = 5. Favorable outcome = 1 (card B). P(B) = 1/5 = 0.2.

Topic 5: Probability involving choosing from objects that are not distinct
Problem 1: A bag has 4 red and 3 blue identical balls. Find the probability of picking a blue ball.
Answer: Total balls = 7. Blue balls = 3. P(blue) = 3/7 ≈ 0.429.

Problem 2: A jar contains 5 white and 2 black marbles. Calculate P(white).
Answer: Total marbles = 7. White marbles = 5. P(white) = 5/7 ≈ 0.714.

Topic 6: Probability of selecting one card from a standard deck
Problem 1: A card is drawn from a standard 52-card deck. Find the probability of drawing a heart.
Answer: Total cards = 52. Hearts = 13. P(heart) = 13/52 = 1/4 = 0.25.

Problem 2: Find the probability of drawing a king from a standard deck.
Answer: Total cards = 52. Kings = 4. P(king) = 4/52 = 1/13 ≈ 0.077.

Topic 7: Finding probabilities of events and complements
Problem 1: A die is rolled. Find the probability of rolling a number less than 4 and its complement.
Answer: Sample space = {1, 2, 3, 4, 5, 6}. Event (less than 4) = {1, 2, 3}, P(less than 4) = 3/6 = 0.5. Complement (4 or more) = {4, 5, 6}, P(complement) = 3/6 = 0.5.

Problem 2: A card is drawn from a deck. Find P(spade) and P(not a spade).
Answer: Spades = 13, total cards = 52. P(spade) = 13/52 = 0.25. P(not a spade) = 1 - 0.25 = 0.75.

Topic 8: Experimental and theoretical probability
Problem 1: A coin is flipped 10 times, landing heads 6 times. Find the experimental probability of heads and compare to the theoretical probability.

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Answer: Experimental P(heads) = 6/10 = 0.6. Theoretical P(heads) = 1/2 = 0.5. Experimental is higher due to small sample size.

Problem 2: A die is rolled 20 times, with 4 appearing 5 times. Calculate experimental P(4) and compare to theoretical.
Answer: Experimental P(4) = 5/20 = 0.25. Theoretical P(4) = 1/6 ≈ 0.167. Experimental is higher, likely due to random variation.

Topic 9: Outcomes and event probability
Problem 1: A spinner with 3 equal sections (red, blue, green) is spun. Find the probability of landing on blue or green.
Answer: Total outcomes = 3. Favorable outcomes = {blue, green} = 2. P(blue or green) = 2/3 ≈ 0.667.

Problem 2: A bag has 2 red, 3 blue balls. Find P(red or blue).
Answer: Total balls = 5. Red or blue = 2 + 3 = 5. P(red or blue) = 5/5 = 1.

Topic 10: Identifying independent events given descriptions of experiments
Problem 1: Are flipping a coin and rolling a die independent events? Explain.
Answer: Yes, independent. The outcome of the coin flip does not affect the die roll, and vice versa.

Problem 2: Are drawing a card, replacing it, and drawing again independent? Justify.
Answer: Yes, independent. Replacing the card ensures the second draw's probabilities are unaffected by the first.

Topic 11: Calculating relative frequencies in a contingency table

Topic 12: Probabilities of draws with replacement
Problem 1: A bag has 3 red, 2 blue balls. Draw one, replace it, draw again. Find P(red, then blue).
Answer: P(red) = 3/5, P(blue) = 2/5 (replacement keeps probabilities same). P(red, then blue) = (3/5) × (2/5) = 6/25 = 0.24.

Problem 2: A deck has 4 aces, 48 non-aces. Draw a card, replace it, draw again. Find P(ace, then non-ace).
Answer: P(ace) = 4/52 = 1/13, P(non-ace) = 48/52 = 12/13. P(ace, then non-ace) = (1/13) × (12/13) = 12/169 ≈ 0.071.

Topic 13: Finding the odds in favor and against
Problem 1: A die is rolled. Find the odds in favor and against rolling a 5.
Answer: P(5) = 1/6, P(not 5) = 5/6. Odds in favor = P(5)/P(not 5) = (1/6)/(5/6) = 1:5. Odds against = P(not 5)/P(5) = 5:1.

Problem 2: A bag has 2 red, 3 blue balls. Find odds in favor and against picking red.
Answer: P(red) = 2/5, P(not red) = 3/5. Odds in favor = (2/5)/(3/5) = 2:3. Odds against = (3/5)/(2/5) = 3:2.

Topic 14: Probability of independent events involving a standard deck of cards
Problem 1: Draw a card from a deck, replace it, draw again. Find P(heart, then heart).
Answer: P(heart) = 13/52 = 1/4. Since independent (replacement), P(heart, then heart) = (1/4) × (1/4) = 1/16 = 0.0625.

Problem 2: Draw a card, replace it, draw again. Find P(ace, then king).
Answer: P(ace) = 4/52 = 1/13, P(king) = 4/52 = 1/13. P(ace, then king) = (1/13) × (1/13) = 1/169 ≈ 0.0059.

Topic 15: Probability of dependent events involving a standard deck of cards
Problem 1: Draw two cards from a deck without replacement. Find P(heart, then heart).
Answer: P(first heart) = 13/52 = 1/4. P(second heart | first heart) = 12/51 (12 hearts left, 51 cards total). P(both hearts) = (1/4) × (12/51) = 12/204 = 1/17 ≈ 0.0588.

Problem 2: Draw two cards without replacement. Find P(ace, then king).
Answer: P(first ace) = 4/52 = 1/13. P(king | ace) = 4/51 (4 kings left, 51 cards). P(ace

Answer: P (first ace) = 4/52 = 1/13. P (king | ace) = 4/51 (kings left, 51 cards). P (ace, then king) = (1/13) × (4/51) = 4/663 ≈ 0.006.