Statistics / Module 12 - Binomial

Topic 1: Binomial problems: Mean and standard deviation
Problem 1: A binomial experiment has 10 trials with a success probability of 0.4. Find the mean and standard deviation.
Answer: Mean = n × p = 10 × 0.4 = 4. Standard deviation = √(n × p × (1 - p)) = √(10 × 0.4 × 0.6) = √2.4 ≈ 1.55.

Problem 2: For a binomial experiment with 20 trials and success probability 0.3, calculate the mean and standard deviation.
Answer: Mean = 20 × 0.3 = 6. Standard deviation = √(20 × 0.3 × 0.7) = √4.2 ≈ 2.05.

Topic 2: Using the binomial formula to find the probability of exactly m successes
Problem 1: A coin is flipped 5 times, with P(head) = 0.5. Find the probability of exactly 3 heads using the binomial formula.
Answer: Formula: P(X = m) = C(n, m) × p^m × (1 - p)^(n-m). Here, n = 5, m = 3, p = 0.5. P(X = 3) = C(5, 3) × (0.5)^3 × (0.5)^2 = 10 × 0.125 × 0.25 = 10 × 0.03125 = 0.3125.

Problem 2: A test has 4 questions, each with P(correct) = 0.2. Find the probability of exactly 2 correct answers.
Answer: n = 4, m = 2, p = 0.2. P(X = 2) = C(4, 2) × (0.2)^2 × (0.8)^2 = 6 × 0.04 × 0.64 = 6 × 0.0256 = 0.1536.

Topic 3: Using the binomial formula to find the probability of more or less than m successes

Topic 4: Binomial problems: Advanced
Problem 1: A machine produces defective items with probability 0.1. In 10 items, find the probability of at least 2 defects.
Answer: P(X ≥ 2) = 1 - [P(X = 0) + P(X = 1)]. n = 10, p = 0.1, q = 0.9. P(X = 0) = C(10, 0) × (0.1)^0 × (0.9)^10 = 1 × 1 × 0.348678 ≈ 0.3487. P(X = 1) = C(10, 1) × (0.1)^1 × (0.9)^9 = 10 × 0.1 × 0.38742 ≈ 0.3874. P(X ≥ 2) = 1 - (0.3487 + 0.3874) = 1 - 0.7361 ≈ 0.2639.

Problem 2: A quiz has 5 true/false questions, with P(correct) = 0.6. Find the probability of getting 3 or 4 correct answers.
Answer: P(X = 3 or 4) = P(X = 3) + P(X = 4). n = 5, p = 0.6, q = 0.4. P(X = 3) = C(5, 3) × (0.6)^3 × (0.4)^2 = 10 × 0.216 × 0.16 = 0.3456. P(X = 4) = C(5, 4) × (0.6)^4 × (0.4)^1 = 5 × 0.1296 × 0.4 = 0.2592. P(X = 3 or 4) = 0.3456 + 0.2592 = 0.6048.

Different Binomials Scenarios:

$P(B) = {}_nC_x\, p^x\, q^{n-x}$
n = total number of trials
x = number of success
${}_nC_x$ = total number of combinations
p = probability of success
q = probability of failures (1 - p)
Formula in words: total number of combinations times probability of success to the number of success times probability of failure to the number of failures

Look below at the 4 different scenarios you will run into. *Note*: the circled part is what we want to find.

n = 8 p = 0.65 **x = exactly 3**

0 1 2 (3) 4 5 6 7 8

$P(B) = {}_nC_x\, p^x\, q^{n-x}$ in calculator: ${}_8C_3\ 0.65^3\ 0.35^5$

n = 8 p = 0.65 **x = 5 or less**

(0 1 2 3 4 5) 6 7 8

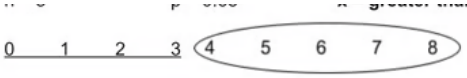
In ALEKS calculator: $p(B_{8,0.65} \leq 5)$

n = 8 p = 0.65 **x = less than 5**

(0 1 2 3 4) 5 6 7 8

In ALEKS calculator: $p(B_{8,0.65} \leq 4)$

n = 8 p = 0.65 **x = greater than 3**



The underlined is what you find in the ALEKS calculator, then you do 1 minus to flip it to the right side:

$1 - p(B_{8,0.65} \leq 3)$

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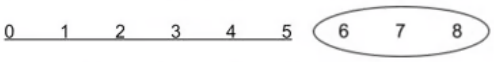
n = 8 p = 0.65 **x = 6 or more**



The underlined is what you find in the ALEKS calculator, then you do 1 minus to flip it to the right side:

$1 - p(B_{8,0.65} \leq 5)$