Statistics /
Module 11 - Discrete Probability Distributions

Topic 1: Making reasonable inferences based on proportion statistics
Problem 1: A survey of 100 people shows 60 prefer brand A. Infer the likely proportion preferring brand A in the population and explain.
Answer: Sample proportion = 60/100 = 0.6 (60%). Inference: Approximately 60% of the population prefers brand A, with a margin of error of ±5-10% for a 95% confidence interval in a large population, assuming random sampling. The sample size supports a reliable estimate, but variability exists due to sampling error.

Problem 2: In a poll of 200 voters, 120 support candidate X. Make an inference about the population proportion and justify.
Answer: Sample proportion = 120/200 = 0.6 (60%). Inference: About 60% of the voter population likely supports candidate X, with a margin of error around ±7% (95% confidence, approximate). The large sample size strengthens the inference, but non-random sampling could introduce bias.

Topic 2: Identifying elements of sets for a real-world situation
Problem 1: In a class, define set A as students who play soccer (10 students) and set B as students who play basketball (8 students). If 3 students play both, identify the elements of A ⬚ B and A ⬚ B.
Answer: A ⬚ B = {students playing both soccer and basketball} = 3 students. A ⬚ B = {students playing soccer or basketball or both} = 10 + 8 - 3 = 15 students.

Problem 2: A club has set C = {members who like hiking, 12 members} and set D = {members who like swimming, 10 members}, with 4 liking both. Identify the elements of C ⬚ D and C ⬚ D.
Answer: C ⬚ D = {members who like both hiking and swimming} = 4 members. C ⬚ D = {members who like hiking or swimming or both} = 12 + 10 - 4 = 18 members.

Topic 3: Probabilities of an event and its complement
Problem 1: A die is rolled. Find the probability of rolling a number less than 4 and its complement.
Answer: Sample space = {1, 2, 3, 4, 5, 6}. Event (less than 4) = {1, 2, 3}, P(less than 4) = 3/6 = 0.5. Complement (4 or more) = {4, 5, 6}, P(complement) = 3/6 = 0.5.

Problem 2: A card is drawn from a 52-card deck. Find P(drawing a heart) and P(not a heart).
Answer: P(heart) = 13/52 = 0.25. P(not a heart) = 1 - 0.25 = 0.75.

Topic 4: Discrete probability distribution: Basic

Topic 5: Discrete probability distribution: Word problem involving cumulative probabilities
Problem 1: A machine produces defects with P(defect) = 0.1 in 5 trials. Find the probability of at most 1 defect.
Answer: n = 5, p = 0.1, q = 0.9. P(X ≤ 1) = P(X = 0) + P(X = 1). P(X = 0) = C(5,0) × (0.1)^0 × (0.9)^5 = 1 × 1 × 0.59049 = 0.59049. P(X = 1) = C(5,1) × (0.1)^1 × (0.9)^4 = 5 × 0.1 × 0.6561 = 0.32805. P(X ≤ 1) = 0.59049 + 0.32805 = 0.91854 ≈ 0.9185.

Problem 2: A quiz has 3 questions, P(correct) = 0.4. Find the probability of at least 2 correct answers.
Answer: n = 3, p = 0.4, q = 0.6. P(X ≥ 2) = P(X = 2) + P(X = 3). P(X = 2) = C(3,2) × (0.4)^2 × (0.6)^1 = 3 × 0.16 × 0.6 = 0.288. P(X = 3) = C(3,3) × (0.4)^3 × (0.6)^0 = 1 × 0.064 × 1 = 0.064. P(X ≥ 2) = 0.288 + 0.064 = 0.352.

Topic 6: Introduction to expectation
Problem 1: A game gives $5 for a win (P = 0.3) and $0 for a loss. Find the expected value of one play.
Answer: E(X) = Σ(x × P(x)). Outcomes: $5 (P = 0.3), $0 (P = 0.7). E(X) = (5 × 0.3) + (0 × 0.7) = 1.5 + 0 = $1.50.

Problem 2: A raffle gives $10 (P = 0.2) or $0 (P = 0.8). Calculate the expected value.

Answer: E(X) = (10 × 0.2) + (0 × 0.8) = 2 + 0 = $2.00.

Topic 7: Computing expected value in a business application
Problem 1: A store sells a product: 50% chance of selling 10 units ($100 profit each), 30% chance of 5 units, 20% chance of 0 units. Find the expected profit.
Answer: Profit per unit = $100. E(X) = (10 × 100 × 0.5) + (5 × 100 × 0.3) + (0 × 100 × 0.2) = 500 + 150 + 0 = $650.

Problem 2: A vendor has a 40% chance of selling 20 items ($5 profit each), 40% chance of 10 items, 20% chance of 0 items. Compute the expected profit.
Answer: E(X) = (20 × 5 × 0.4) + (10 × 5 × 0.4) + (0 × 5 × 0.2) = 40 + 20 + 0 = $60.