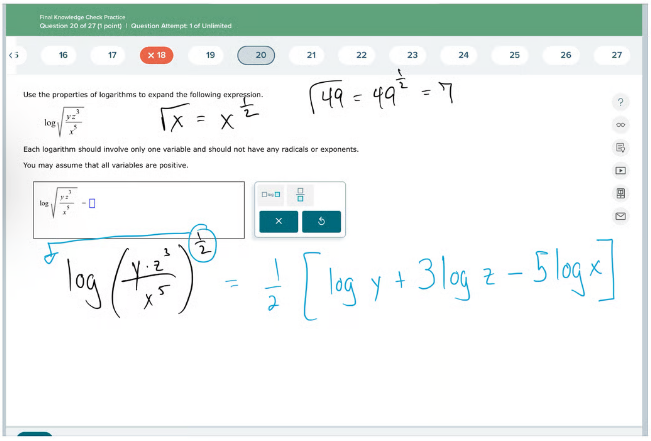
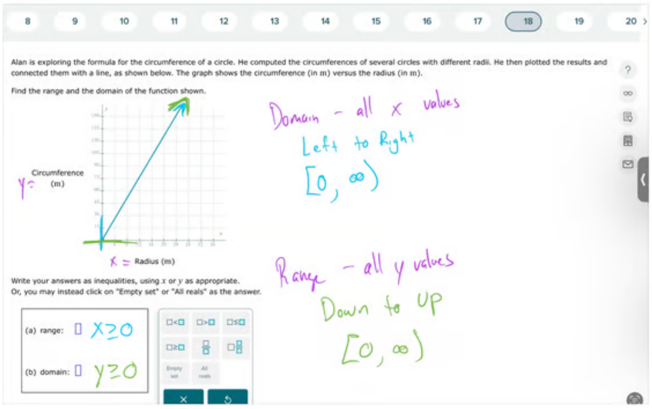
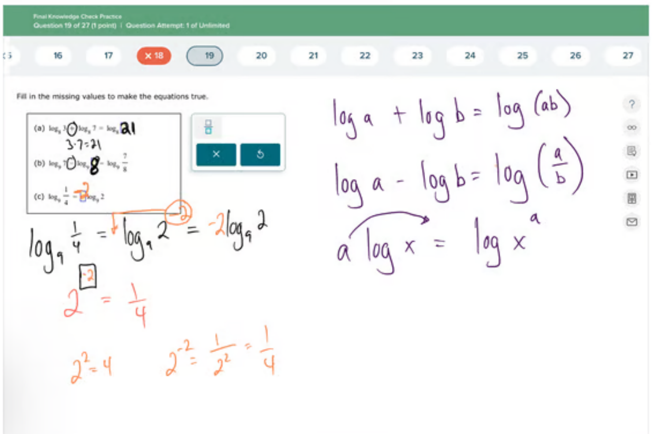
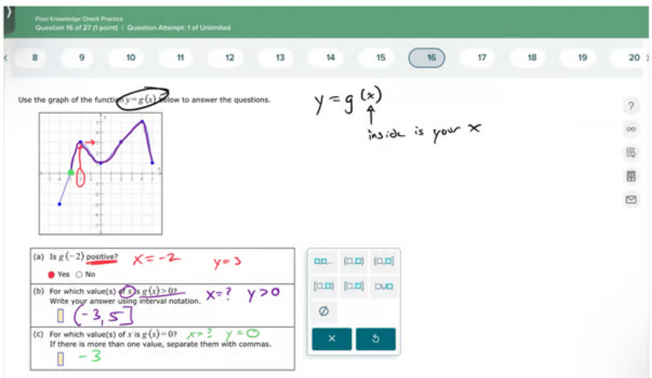
Algebra Reviews /

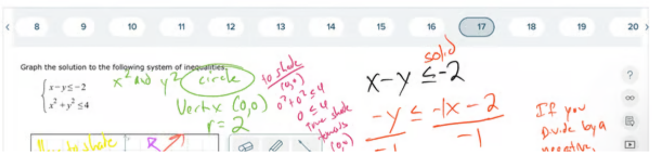
1105 (2024 Fall) Final Knowledge Check Practice

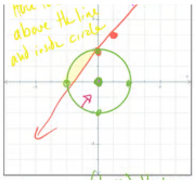












above the line and inside circle

Circle (h,k) Vertex r=radius

$(x-h)^2+(y-k)^2=r^2$

$y \geq x+2$ FLIP THE SIGN

$m=1=\frac{1}{1}\ \frac{\text{up 1}}{\text{right 1}}$

$b: 2$ start at $(0,2)$

to shade
Plug in a point not on the line

$(0,0)$ $0-0 \leq -2$
$0 \leq -2$ False shade away from $(0,0)$

2 differences
① $<, >$ dashed
$\leq, \geq$ solid
② shade

8 9 10 11 12 13 14 15 16 17 18 19 20

For the real-valued functions $f(x)=\frac{x+4}{x-3}$ and $g(x)=\boxed{2x+9}$ find the composition $f \circ g$ and specify its domain using interval notation.

$(f \circ g)(x) = \square$ $\frac{2x+13}{2x+6}$

Domain of $f \circ g$: $\square$

$(f \circ g)(x) = f(g(x)) = f(2x+9)$

$= \frac{(2x+9)+4}{(2x+9)-3} = \boxed{\frac{2x+13}{2x+6}}$

4 terms
only divide if you can divide all 4 by 2

Domain
figure out what #'s x cannot be
Can't divide by 0
Set denom $=0$

$2x+6=0$
$2x=-6$
$\boxed{x \neq -3}$

-3

Domain: $(-\infty, -3) \cup (-3, \infty)$

8 9 10 11 12 13 14 15 16 17 18 19 20

A wire that is 48 centimeters long is shown below.
The wire is cut into two pieces, and each piece is bent and formed into the shape of a square.

Suppose that the side length (in centimeters) of one square is x, as shown below.

48 cm

x $A=x^2$ x x x $12-x$ $A=(12-x)^2$

$\frac{48}{4}=12$ 12 12 12 12

Area of square
$A=s^2$

(a) Find a function that gives the total area $A(x)$ enclosed by the two squares (in square centimeters) in terms of x.

$A(x) = \square$

(b) Find the side length x that minimizes the total area of the two squares.

Side length x : $\square$ centimeters 2

(c) What is the minimum area enclosed by the two squares?

Minimum area: $\square$ square centimeters 104

$x^2+(12-x)^2$
FOIL
$(12-x)(12-x)$

$144-12x$
$-12x+x^2$
$144-24x+x^2$

$x^2+144-24x+x^2$

ⓐ $A(x)=2x^2-24x+144$

Since positive ⋃ min

Vertex $(2, 104)$

Different example
$-3x^2+5x-2$
down b/c negative ⋂ max

To find Vertex:
$\left(\frac{-b}{2a}, \text{Plug it in}\right)$

$a=2$ $b=-24$ $c=144$

$\frac{24}{2(2)}=\boxed{\frac{x}{6}}$

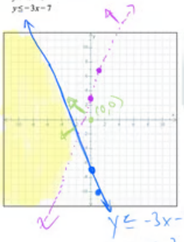
$A(2)=2(2)^2-24(2)+144$
$8 - 48 + 144$
$A(2)=104$

Final Knowledge Check Practice
Question 12 of 27 (1 point) | Question Attempt: 1 of Unlimited

1 2 3 4 5 6 7 8 9 10 × 11 12 13

Graph the solution to the following system of inequalities.

$y > 4x+3$
$y \leq -3x-7$



$y > 4x+3$
$m=4$ $\frac{4}{1}$ $\frac{\text{up 4}}{\text{right 1}}$
$b=3$ start here

$0 > 4(0)+3$
$0 > 3$ No
Shade Away $(0,0)$

① $>, <$ dashed
$\leq, \geq$ solid
② shade

$y \leq -3x-7$
$m=-3$ $\frac{-3}{1}$ $\frac{\text{down 3}}{\text{right 1}}$
$b=-7$ start here

$0 \leq -3(0)-7$
$0 \leq -7$ No shade away from $(0,0)$

Check

Final Knowledge Check Practice
Question 13 of 27 (1 point) | Question Attempt: 1 of Unlimited

1 2 3 4 5 6 7 8 9 10 × 11 12 13

Suppose that the functions f and g are defined as follows.

$f(x)=\frac{1}{5x^2+3}$
$g(x)=\sqrt{3x+4}$

Find $f \cdot g$ and $f+g$. Then, give their domains using interval notation.

$(f \cdot g)(x) = \square$

Domain of $f \cdot g$: $\square$

$(f+g)(x) = \square$

Domain of $f+g$: $\square$

$(f \cdot g)(x) = f(x) \cdot g(x)$

$\frac{1}{5x^2+3} \cdot \frac{\sqrt{3x+4}}{1}$

$\boxed{\frac{\sqrt{3x+4}}{5x^2+3}}$

Domain: $5x^2+3=0$
$5x^2=-3$
$x^2=-\frac{3}{5}$
$x=\pm\sqrt{-\frac{3}{5}}$
imaginary number
$(-\infty, \infty)$

$\sqrt{3x+4}$
$3x+4 \geq 0$
$3x \geq -4$
$x \geq -\frac{4}{3}$
$\left[-\frac{4}{3}, 0\right)$

$(f+g)(x) = f(x)+g(x)$
$\boxed{\frac{1}{5x^2+3}+\sqrt{3x+4}}$

Domain
$\left[-\frac{4}{3}, 0\right)$

Check

1 2 3 4 5 6 7 8 9 10 11 12 13

D and E are sets of real numbers defined as follows.

$D = \{v \mid v \geq 4\}$

$E = \{v \mid v < 8\}$

Write $D \cap E$ and $D \cup E$ using interval notation.
If the set is empty, write $\varnothing$.

$D \cap E =$ ▯

$D \cup E =$ ▯

$\cup$ Union "or" bring together

$\cap$ intersection "and" share

$D = \{4, 5, 6, 7, \dots\}$

$E = \{\dots, 4, 5, 6, 7\}$

Intersect

$D \cap E = [4, 7]$

"and" share

$D \cup E = (-\infty, \infty)$

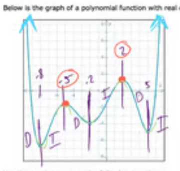
"or" bring together

Anywhere the number line is green OR blue which is everywhere

Check

1 2 3 4 5 6 7 8 9 10 11 12 13

Below is the graph of a polynomial function with real coefficients. All local extrema of the function are shown in the graph.



both up = LC Positive

Use the graph to answer the following questions.

(a) Over which intervals is the function decreasing? Choose all that apply.

■ $(-\infty, -8)$ ■ $(-5, -2)$ □ $(-2, 2)$ ■ $(2, 5)$ □ $(-2, 5)$ □ $(5, \infty)$

(b) At which x-values does the function have local maxima? If there is more than one value, separate them with commas.

▯ -5, 2

(c) What is the sign of the function's leading coefficient?

(Choose one) Positive

(d) Which of the following is a possibility for the degree of the function? Choose all that apply.

□ 4 □ 5 ■ 6 ■ 7 ■ 8 ■ 9

count # of turns = 5

max # of turns = deg − 1

Check

degree - highest exponent

1 2 3 4 5 6 7 8 9 10 11 12 13

Graph the rational function.

$f(x) = \frac{6}{2x-1}$

① VA set denom = 0

$2x - 1 = 0$

$2x = 1$

$x \neq \frac{1}{2}$

② HA

$\frac{\text{deg numerator}}{\text{deg denominator}}$ $\frac{\text{deg: 0}}{\text{deg: 1}}$ num < den $y = 0$

Start by drawing the vertical and horizontal asymptotes. Then plot two points on each piece of the graph. Finally, click on the graph-a-function button.

deg num < deg den $\boxed{y = 0}$

deg num = deg den $\frac{\text{LC num}}{\text{LC den}}$

Ex $\frac{\boxed{2}x^3 + 5}{\boxed{3}x^3 - 2}$ $\boxed{y = \frac{2}{3}}$

deg num > deg den No HA

③ Right of VA:

$x = 1$

$f(1) = \frac{6}{2(1)-1} = \frac{6}{1} = 6$ $(1, 6)$

$x = 2$

$f(2) = \frac{6}{2(2)-1} = \frac{6}{3} = 2$ $(2, 2)$

left of VA:

$x = 0$

$f(0) = \frac{6}{2(0)-1} = \frac{6}{-1} = -6$ $(0, -6)$

$x = -1$

$f(-1) = \frac{6}{2(-1)-1} = \frac{6}{-3} = -2$ $(-1, -2)$

Check

1 2 3 4 5 6 7 8 9 10 11 12 13

Suppose that the functions u and w are defined as follows.

$u(x) = -2x - 1$

$w(x) = 2x^2 + 1$

Find the following.

$(w \circ u)(-1) =$ ▯ 3

$(u \circ w)(-1) =$ ▯ -7

$(w \circ u)(-1) = w(u(-1))$

$u(-1) = -2(-1) - 1 = 1$

$w(1) = 2(1)^2 + 1 = \boxed{3}$

$(u \circ w)(-1) = u(w(-1))$

$w(-1) = 2(-1)^2 + 1 = 3$

$u(3) = -2(3) - 1 = \boxed{-7}$

Check

$x^2 - 18x + 81$

+81

-9 -9
-18

$\boxed{(x-9)(x-9)}$

FOIL back out to check answer

or $(x-9)^2$

$-4 + \sqrt{u-8} = -3$

$+4 \qquad\qquad +4$

Plug back in to check answer

$-4 + \sqrt{9-8} = -3$

$\sqrt{1}$

$-4 + 1 = -3$ ✓

$(\sqrt{u-8})^2 = 1^2$

$u - 8 = 1$

$+8 \qquad +8$

$\boxed{u = 9}$

$\left(\dfrac{m^6 n^3}{m^3 n^5}\right)^5$

m: $6-3=3$ positive top

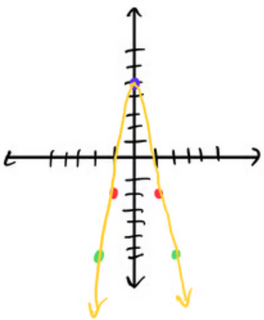
$n = 3-5 = -2$ negative bottom

$\dfrac{m^3}{n^2}$ OR $\dfrac{mmmmmm\ nnn}{mmm\ nnnnn}$ $\dfrac{m^3}{n^2}$

$\left(\dfrac{m^3}{n^2}\right)^5$ $\boxed{\dfrac{m^{15}}{n^{10}}}$

$h(x) = -3x^2 + 5$

Graph one point with $x=0$, two negative points, two positive points



x	y
-2	-7
-1	2
0	5
1	2
2	-7

use symmetry

$h(-2) = -3(-2)^2 + 5$
$-3(4) + 5$
$-12 + 5$
$h(-2) = -7$

$h(-1) = -3(-1)^2 + 5$
$-3(1) + 5$
$-3 + 5$
$h(-1) = 2$

$h(0) = -3(0)^2 + 5$
$-3(0) + 5$
$h(0) = 5$

$\log_5(2x-5) = 2$

$5^2 = 2x - 5$

$25 = 2x - 5$

$+5 \qquad +5$

$\dfrac{30}{2} = \dfrac{2x}{2}$

$\boxed{x = 15}$

Plug in to check answer

$\log_5(2(15) - 5) = 2$

$30 - 5$

25

$\log_5(25) = 2$

$5^2 = 25$ ✓

How to convert log:

$3^x = 27$

$\log_3 27 = x$

$\log_5 x = 2$

$5^2 = x$

$f(x) = -x^2 + x + 6$

x-int: $-2, 3$

y-int: 6

Xint $y = 0$
Set $= 0$

Don't want front # to be negative

$0 = -x^2 + x + 6$
$\quad -1$

$0 = x^2 - 1x - 6$

-6, $+2$, -3, -1

$0 = (x+2)(x-3)$

$x+2=0 \qquad x-3=0$

$x=-2 \qquad x=3$

Yint $x = 0$

$f(0) = -(0)^2 + (0) + 6$

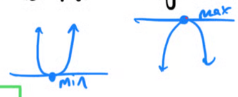
$f(0) = 6$

$h(t) = 120t - 16t^2$

$-16t^2 + 120t + 0$

$a = -16 \quad b = 120 \quad c = 0$

what is max height?



Vertex
$\left(\frac{-b}{2a}, \text{plug it in}\right)$

find vertex max

$\frac{-120}{2(-16)} = \frac{-120}{-32} = \boxed{\overset{t}{3.75 \text{ sec}}}$

3.75 seconds to reach max height

Plug in $t = 3.75$

$h(3.75) = 120(3.75) - 16(3.75)^2$

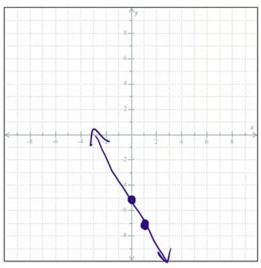
$450 - 225$

$h(3.75) = \boxed{225 \text{ ft max height}}$

---

15 16 17 18 19 20 21 22 23 24 25 26 2

Graph the function $g(x) = -2x - 5$.



$g(x) = -2x - 5$

$y = mx + b$

$m = -2 = \frac{-2}{1}$ $\frac{\text{down } 2}{\text{right } 1}$

$b = -5$ start at $(0, -5)$

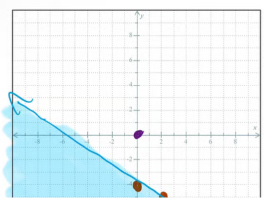
---

15 16 17 18 19 20 21 22 23 24 25 ✕ 26 2

Graph the inequality.

$y \le -\frac{1}{2}x - 4$

to shade
plug in point not on the line
$(0,0)$ $0 \le -\frac{1}{2}(0) - 4$
$0 \le -4$ False
shade away from $(0,0)$



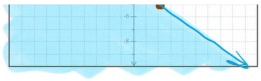
solid

$y \le -\frac{1}{2}x - 4$

$y = mx + b$

$m = -\frac{1}{2}$ $\frac{\text{down } 1}{\text{right } 2}$

$b = -4$ Start at $(0, -4)$



2 differences!

1. $<, >$ dashed $\le, \ge$ solid
2. shade

$$\ln 7^{\boxed{y-3}} = \ln 9$$

$$(y-3)\ln 7 = \ln 9$$

$$y\ln 7 - 3\ln 7 = \ln 9$$
$$+3\ln 7 \qquad 3\ln 7$$

$$\frac{y\ln 7}{\ln 7} = \frac{\ln 9 - 3\ln 7}{\ln 7}$$

$$\boxed{y = \frac{\ln 9 - 3\ln 7}{\ln 7} = -1.871}$$

Exact = in terms ln   Approx = decimal

$\log_2 8 = 3 \qquad 2^3 = 8$

$\log_e y = x \qquad e^x = y$

$e = 2.718....$

$\ln y = x \qquad e^x = y$

the Natural log

$(x-9)(x-9)(x+11)(x-5)(x-5)$

$$f(x) = 8(x-9)^2(x+11)(x-5)^2$$

opposite

$x-9=0 \quad x+11=0 \quad x-5=0$

$x=9 \quad x=-11 \quad x=5$

two of those   two of those

look at the exponent to see multiplicity

Zero(s) of multiplicity one: -11

Zero(s) of multiplicity two: $9, 5$

Zero(s) of multiplicity three:
